AMYGDALA $z \mapsto z^2+c$

A Newsletter of Fractals & $\mathcal{M}$ (the Mandelbrot Set)

Issue #23

May 17, 1991

IN THIS ISSUE...

...we have two articles, written independently, on closely related topics; each a critique or expansion of the ideas which appeared in *A Journey to the West*, by Esseh N. Namreh (Amygdala #5, July, 1987). In that article, Namreh discussed two among the infinity of Mandelbrot set "midgets" which occur along the spike (and may in fact *constitute* it!): the Biggest Midget and the John Dewey Jones Midget.

In their article, *The Uttermost West Revisited*, A.G. Davis Philip and Adam Robucci give some results of an intensive investigation of midgets, and exhibit the result of an incredibly deep penetration into the Mandelbrot set: their "Cycle 250" midget, with a magnification of $9.05{\times}10^{298}$.

In *A "Journey to the West" Revisited*, Kerry Mitchell takes Namreh to task for supposedly supposing that the singularity at -2.0 consists of *nothing but* "endless Yin/Yang loops spinning endlessly inward and eastward", whereas in fact there are an infinite number of midgets near it. Mitchell discusses these midgets, their location, size, and "reach": how close the contour line associated with the midget comes to the real axis.

The first three slides in the supplement to this issue have been chosen to edify rather than to please: they show in color the incredibly tiny midgets of Cycle 85, 200, and 250 discussed in the Philip/Robucci article.

THE SLIDES (S23)

4000 was produced by Dave Platt, who says of it: This image, "Ken's N–spiral", is centered at $-0.7623{,}8220{,}3158 + 0.0955{,}6345{,}5061i \times 1.66{\times}10^{10}$. It is one of the many spirals that Ken Philip has been exploring during the last few months. Pixelation = 1500 × 1000; calculated using the distance-estimator algorithm in MandelZot 2.0. Colored using a palette of bright pastels, saved as a PICT file, and transcribed to film on an Agfa Matrix slidemaker by the good folks at PhotoTime in Palo Alto.

819, **820**, and **821** appear in greyscale as illustrations in *The Uttermost West: Revisited* by A.G. Davis Philip and Adam Robucci, in this issue. They are the high-magnification midgets of Cycle 250, 200, and 85, respectively. Please don't be put off by the ultra-low resolution! Considering that #819 has magnification $9.05{\times}10^{298}$, it's a miracle that the authors were able to compute it at all!

CONTENTS

THE UTTERMOST WEST: REVISITED

A. G. Davis Philip and Adam Robucci, Union College

1. INTRODUCTION

In Amygdala 5, p. 2, an article, by Namreh, entitled "A Journey to the West", displayed pictures of the end of the spike of the Mandelbrot Set, in an area called the "Uttermost West" at a maximum magnification of 70,000. One of the pictures showed the John Dewey Jones midget at a real value = -1.99638, also at a magnification of 70,000. A comparison of the end of the Mandelbrot Set Spike at a magnification of 4,400 with that at 70,000 showed that the pattern of the loops was the same in each picture. At each of the intersections of a loop with the spike axis, if the picture is magnified sufficiently, a midget (a small, but not exact, replica of the Mandelbrot Set) can be found.

Figure 1 shows a picture of the spike, and a number of midgets can be seen, even at this low magnification. The escape radius was set to 10, so the dwell bands do not show up. Two of the midgets are labeled with their cycle number (Cycle 3 at R = -1.755 and Cycle 4 at R =-1.99638. In Figure 2, three different enlargements of the end of the spike are shown, this time with the escape radius set to 2 so the dwell band structure shows up. In Figure 2a the magnification is 3.9, in 2b it is 13.3 and in 2b it is 155. The dwell bands are numbered by one digit numbers. Points in dwell band 1 escape the set after 1 iteration, in dwell band 2 after two iterations and so on. Cycle 4 to Cycle 7 midgets are located at the points indicated by C4 - C7. The curve of dwell band 4 points to the Cycle 4 midget, and so on. The John Dewey Jones midget is the cycle 6 midget at R = - 1.99638. For each dwell band there is a midget which is the westernmost midget with that cycle number. It has been found that if the magnification window is placed on one of the midgets in this series and the magnification increased, a midget will eventually appear on the left side of the window which will be the largest midget in the window. This midget is the next one in the series. In Amygdala 18, p. 2 there is an article concerning dwell bands by Ken Philip (KWP) giving additional details concerning the dwell bands.

Benoit Mandelbrot came to Union College in 1988 to deliver the "Steinmetz Lecture" and AGDP asked him then what was the highest magnification which he knew of that had been used for a picture of a portion of the Mandelbrot Set. He said that he knew of one group that had produced a picture with a magnification equal to Avogadro's number (6.0249×10^{23}). This remark interested AGDP and KWP, and a search for high-magnification midgets was started. K. W. Philip used the high precision

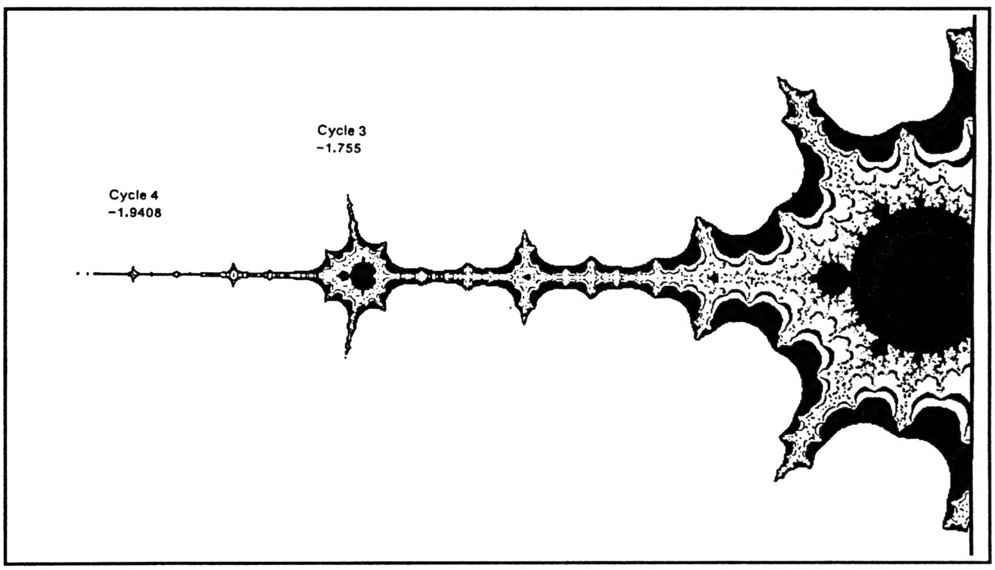


Figure 1. An enlarged view of the spike on the Mandelbrot Set. The real values shown are between -2.076 and -1.22267. Two of the major midgets (Cycles 3 and 4) are indicated.

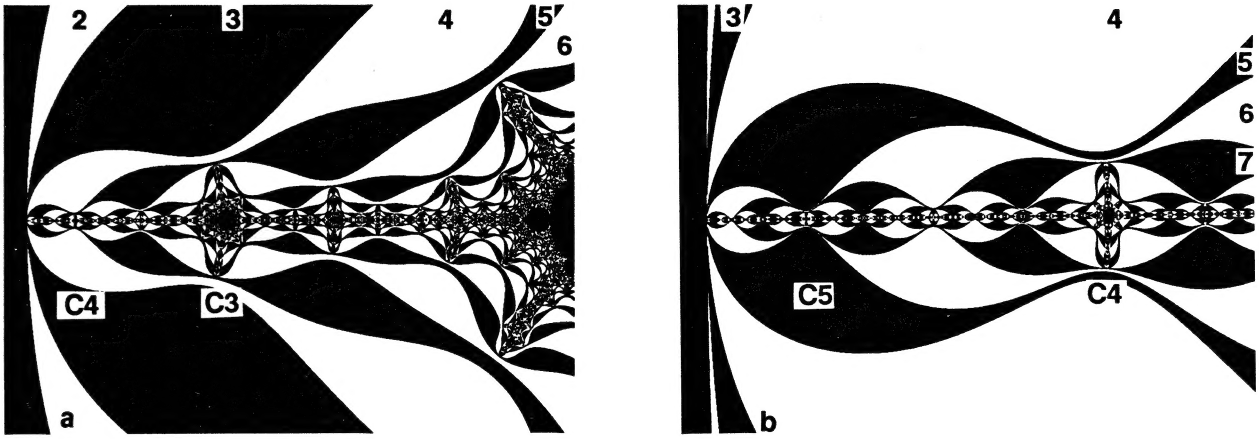
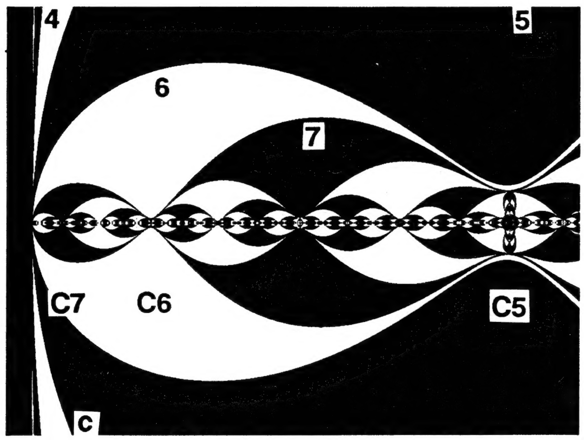


Figure 2. The end of the spike. In 2a the magnification is 3. In 2b it is 13, and in 2c it is 155. The dwell bands are indicated by the numbers 2 - 7 placed in the loops. Midgets of different cycle numbers are indicated by C3 - C7.

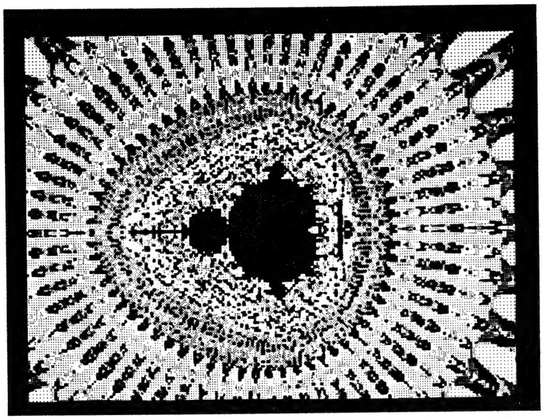


routines in True Basic to write a Mandelbrot program for his Macintosh computer which converted numbers to strings. He produced a series of high-magnification midgets, ending up with the cycle 28 midget at a magnification of 2×10^{31}. This midget was noted in one of Dewdney's (1989) *Computer Recreations* articles in Scientific American. There is an interesting astronomical analog for this midget. If one imagines that the Mandelbrot Set is the size of the Milky Way Galaxy (assuming a radius of 100 Kiloparsecs), then the cycle 28 midget represents one hydrogen atom. The Cycle 28 midget is shown in Fig 3a (computed on the Union College VAX).

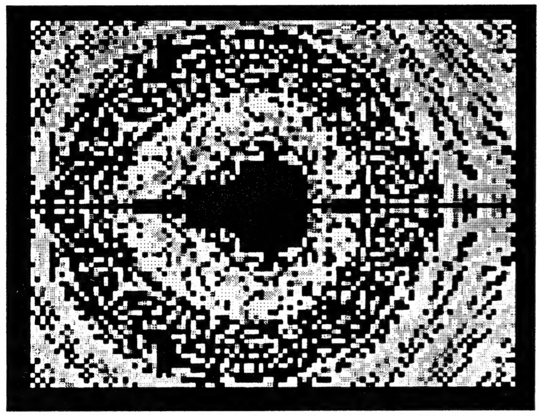
2. THE HIGH MAGNIFICATION MIDGET PROJECT

Mike Frame, a mathematics professor at Union College, teaches a course in fractals each year and some of the students elect to do a term project involving the study of some aspects of the Mandelbrot Set. Last year AR was interested in the project of finding midgets at very high magnification at the very end of the spike. There are two problems that had to be solved. One was to write mathematical routines that would handle numbers of any precision desired. (For example we are now working with numbers having more than 350 digits.) The second was to locate the position of midgets with very high cycle numbers.

The high precision Mandelbrot generation programs are written in ANSI C. They use extended precision arithmetic routines that manipulate numbers as strings, thus giving us unlimited precision. The program iterates the function $F(Z) = Z^2 + C$ until $|Z| > 2$ or a predefined number of iterations has been exceeded. The number of iterations for each point in the plane is then saved in an ASCII file for decoding by SMOOTH.EXE, the viewing program. Since the micro-midgets we are searching for are on the real axis, the program only generates the positive imaginary part of the midgets (the top half of the image), and the viewing program reflects the points about the real axis to produce the final image. Thus, in our small 100×76 pixel images we need only calculate 100×39 pixels.



a



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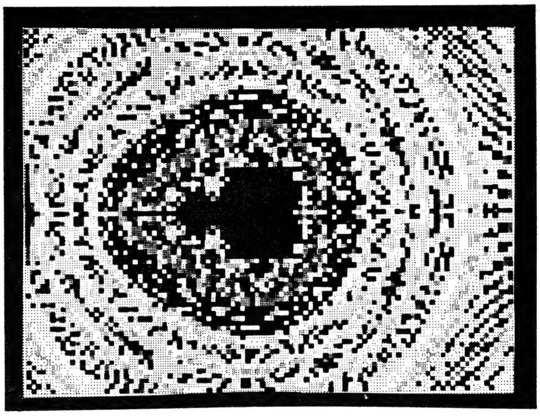

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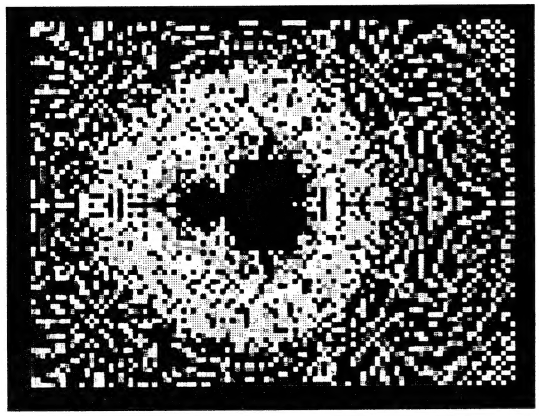

d

Figure 3. 3a shows the Cycle 28 (KWP) midget. 3b shows the Cycle 175 midget, 3c the Cycle 200 midget, and 3d the Cycle 250 midget. The positions and magnifications are given in Table 1. The resolution in 3a is 400 × 300, and in 3b - 3d it is 100 × 76 pixels.

Locating the specific midget with a given cycle number is done with the Cycle Finder program, written in TURBO C. It also uses the extended precision math routines. The cycle finder works on the principle that a stable N–cycle in the Mandelbrot set can be located as a root of a 2^Nth degree equation. The Cycle Finder finds the most negative real root of the equation using the Secant Method. The program is supplied with two initial guesses of where the root is located, the cycle number of the midget we are trying to find, and a small quantity, ε, representing the accuracy of the final approximation. The program then iterates the function $F(Z)$ N times where N is the value of the cycle number. By iterating the function N times it is in fact evaluating a function of degree 2^N. The program keeps iterating the function and refining the estimated root until the difference between the new guess and the old guess is less then ε. The final value output by the program is an approximation of a root of the equation and corresponds to a stable N–cycle in the Mandelbrot Set.

A second program, named "SMOOTH" plots the data files on the screen. It was written in TURBO C++, Version 1. The micro-midgets program writes a file consisting of the iteration value for each point it calculates in the image. These iteration values are used as the color parameter in the display routine. SMOOTH is supplied with an integer N, to be used as the "smooth value". What this does is group dwell bands together in groups of N and assigns them the same color, thus making features easier to identify. Color is assigned to each pixel based on the following: $Color = int(\frac{Iteration}{N})$.

In Word Perfect there is a program called "GRAB" which allows one to capture graphic images on the screen and move them to the WordPerfect area. The color slides were produced by

photographing the display screen, where the small (1 1/4" × 1 3/4") image is brought up as a full-screen image in WordPerfect. To print the black and white images the program Pizzaz Plus (Application Techniques) was used since it permits control of the greyscale in each of the images.

We started with the Cycle 28 (KWP) midget, as a calibration point. Soon we were up to the Cycle 35 midget, which has a magnification of 2×10^{40}. An astronomical analog can be constructed for this midget also. If the Mandelbrot Set is imagined to be the size of the observable universe, (assuming a radius of 3.2×10^{27} cm) then the Cycle 35 midget, on this scale, represents a single proton. By March of 1990 we reached the Cycle 42 midget at 3.2×10^{48}. By June the calculations were taking about a week per one 100 × 76 pixel image. We did the Cycle 175 midget which has a magnification of 4.44×10^{208}. Pictures of the Cycle 42 and 175 midgets were published in Philip 1990, and the Cycle 175 midget is shown here in Figure 3b. A picture of the Cycle 200 midget will be found in Frame et al. (1991) and is shown here in Figure 3c. The smallest midget done to date is the Cycle 250 midget, with a magnification of 9.05×10^{298}. The coordinates, magnification and dwell for each of the midgets pictured in this article are shown in Table 1. Color slides of three of the midgets (Cycle 85, Cycle 200 and Cycle 250) are included with the slide supplement for this issue.

If one scans the midget pictures the midget at the center is always similar to the others. The differences here are due to the low resolution of the 100 × 76 pixel pictures. If a coarse grid is placed on top of the image of a midget and the grid squares colored black and white, the resulting picture will have a slightly different set of black pixels on areas just outside the edge of the midget. Figure 4 shows the Cycle 85 midget at a higher resolution (400 × 300 pixels) and here one can see a much finer midget because of the finer grid. The radicals on the body can be seen clearly to at least radical 6. On the low resolution 100 × 76 pixel pictures, only Radical 3 can be identified.

The Cycle 28 picture shows an outer ring of 64 lobes. Inside this ring are other rings (of 128, 256 etc. lobes) but at this resolution the details of the rings can not be resolved. As one scan the pictures showing the higher magnification midgets the main

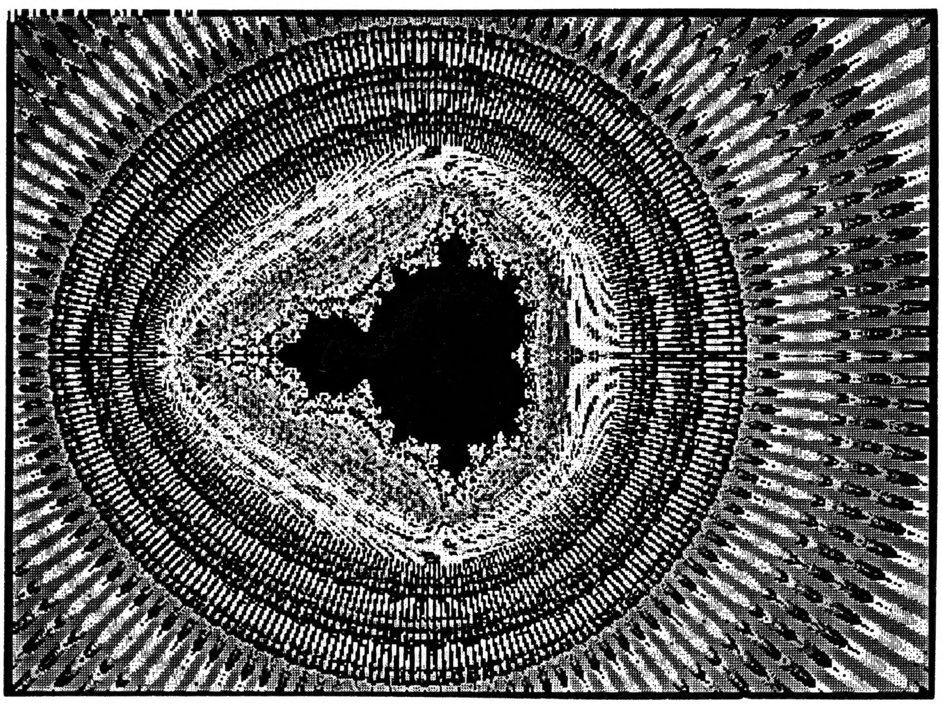

Figure 4. The Cycle 85 midget at a resolution of 400 × 300 pixels.

feature that changes is the density of the packing of the rings. By Cycle 250 the rings are so closely packed that it is hard even to resolve the rings.

There is a book in preparation (entitled *Mm*, by Philip, Robucci, Frame and Philip) which shows all the midgets from Cycle 3 to Cycle 85, then to Cycle 125 by fives, followed by Cycles 150, 175, 200, 225 and 250. Positions, dwells and magnifications are given for each midget. Most of the midgets were calculated on IBM PCs, but some were computed on the Union College VAX. The Cycle 250 midget was calculated on a Zenith PC and one of the Union College's Dec System 5000s (Decoy). In the book details of the high precision routines are given, plus some interesting mathematical relations between midgets on the spike. In the Appendix there is a table of midget positions up to Cycle 350 — the complete file will soon be available on a floppy disk.

Some interesting relationships were found and these have been presented in a paper to be published in *Computers and Graphics* (Frame et al. 1991). If the ratio of distances, is taken, between head N and head N+1, with head N+1 and head N+2, defined as:

$$R_m(n) = \frac{c_n - c_{n+1}}{c_{n+1} - c_{n+2}}$$

```
Cycle 250    Mag = 9.05 x 10^298    Max Dwell = 10,000

R = -1.99999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999
999999999999547734799326345714612849868772624150857945748314942293641602
039247450498736039839970632632438592848265993465823922757562994579389408
98881380801405189335974207058O6

Cycle 200    Mag = 5.63 x 10^238   Max Dwell = 10,000

R = -1.99999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999942668574690370152 4
868873414969921982795393910464951564378620882667385903299117419504340886
237075855598460105829200658465344388138181393730976369034307923437239154
65053321

Cycle 175    Mag = 4.44 x 10^208    Max Dwell = 10,000

R = -1.99999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999354505535847328988700905833931643
745672283161856447381040061547187817860426924771884415486898706134457685
23494248535314832534265151182

Cycle  85    Mag = 1.89 x 10^100   Max Dwell = 2000

R = -1.99999999999999999999999999999999999999999999999999010782612032415
331455059322565655484274959652860994832497865309208641657835650469146973
177816619829086170543

Cycle  28    Mag = 3.51 x 10^31    Max Dwell = 1500

R =  -1.99999999999999979454758656191510
```

Table 1. Positions, magnifications, and dwell settings for the midgets in this paper.

where m is the exponent in the term $Z^m + C$, we find that as n approaches infinity $R_2(n)$ approaches 4 at the limit. If one investigates the behavior of midgets in the equation $Z^4 + C$ we found that R_4 approaches 8, for an exponent of 6, R_6 approaches 12 and so on. This is an example of a new scaling in the Mandelbrot Set. For details, see the paper.

ACKNOWLEDGEMENTS

We have benefitted from the help of many people. First of all discussions with Mike Frame were instrumental in understanding the mathematics involved in this study. Lance Spallholz has allowed us to use some of his computers during the night-time hours to calculate some of the midgets. K. W. Philip and AGDP have had extensive discussions over Bitnet concerning this project.

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A "JOURNEY TO THE WEST" REVISITED

Kerry Mitchell

I was intrigued by Esseh Namreh's article (*A Journey to the West*) in Amygdala #5; I too had wondered exactly what happened on the spike west of the head of the Mandelbrot set. Armed with Crystal Rose's "Analytic Art," my Amiga 1000, and *lots* of patience, I set forth.

As Namreh observed, the contours between regions of constant dwell have a wavy nature near the spike, with the number of waves doubling with each unit increase in dwell. The journey in the article continued to -1.999986, with a magnification of 70,000. At this point, nothing but "endless Yin/Yang loops spinning endlessly inward and eastward" was observed. One of the things that attracted my attention to the Mandelbrot set was its attention to detail at all scales. It thus seemed incongruous to me that, at some point, the midgets should suddenly disappear.

For my work, it was necessary to be able to discuss the "size" of a midget, thus, a length scale was needed. In $\mathcal{M}$, the distance from the cusp (x = 0.25) to the break between the head and the body (x = -0.75) is 1.0. $\mathcal{M}$ was assigned an arbitrary size of 1. Using this analogy with the midgets, the distance along the real axis from the cusp to the head/neck break was found, and this became the "size". The location of each midget was determined in terms of this length scale. For the main set, the origin of the coordinate system (x = 0) is such that the cusp is located 25% of the "size" to the right of the origin. In other words, for $\mathcal{M}$, size = 1, and the cusp is at x = 25% of 1 = 0.25. For a midget, the "origin" was assumed to be 25% of this length left of its cusp.

Another parameter reflected a midget's effect on the rest of the set. My contention is that, whenever a contour line bends near the spike (such as the dip in line D_4 near x = -1.75), it is due to the influence of a midget (such as the "Biggest Midget" at x = -1.75). How close this line gets to the real axis in the region of the midget, I termed the "reach" of the midget. Looking at the miniMandelbrots associated with lines D_4 - D_{10}, they were found to follow a definite set of scaling laws. The approximate empirical relations are:

$$origin = -2 + 60 \cdot 2^{-2 \cdot dwell}$$

$$reach = 300 \cdot 2^{-3 \cdot dwell}$$

$$size = 1000 \cdot 2^{-4 \cdot dwell}$$

The exact numerical values of the parameters are not so important as are the trends, given by the

exponents. This explains why Namreh didn't observe more midgets. These relations imply that, in going from the midget associated with line D_n to that associated with line D_{n+1}, three things happened. First, the new midget is 4 times closer to the point x = -2 than the previous one. Second, the reach of the new midget is 8 times smaller than the reach of the previous one. Finally, the size of the new midget is 1/16 the size of the previous midget. With increasing dwell, the size of each midget falls with the square of its distance from x = -2, meaning that each successive midget is affected less and less by its neighbors. The reach falls with the distance to the 1.5 power, which means that the closer you get to x = -2, the harder you have to look to find any evidence of the midget. The size of the midget falls faster than its reach, so that once the outer extent is found, several orders of magnitude more zoom may be needed to reach the actual midget.

The midget for line D_9 is located at x = –1.99977. With a magnification of 440, Namreh saw nothing but the wavy structure. According to my estimates, the magnification would have to be on the order of 0.5 million to obtain a picture dominated by the "reach" length scale. To capture the actual midget, a zoom of 15 million is required, leading to a picture in which the "size" is 10% of the width of the window. For the D_{11} midget at –1.999986, the "reach" picture requires a zoom of 30 million, and 3 billion for the midget itself.

While I have only looked at a few of the available midgets on the spike, I believe that the scaling laws are indicative of what happens as x approaches -2. It would appear that the wavy structure extends to the edge of the set, with the dip in each dwell line corresponding to a midget. The location, size, and extent of influence of the midgets appear to correspond to simple scaling laws. The existence of simple scaling laws is consistent with the spirit of the Mandelbrot set: self-similarity at all scales, and incredible complexity as a result of very simple nonlinearities.

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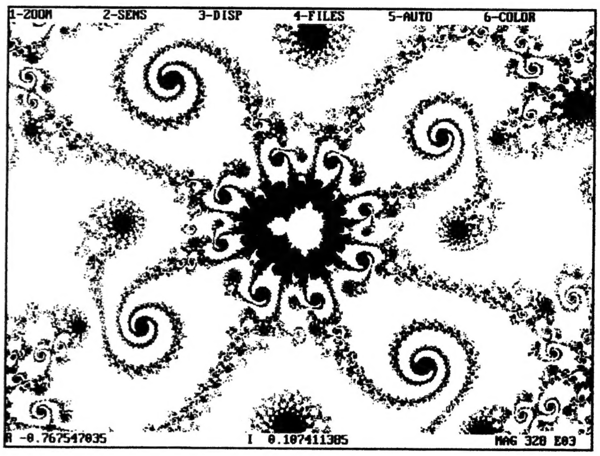

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